U. S. ARMY TEST AND EVALUATION COMMAND
BACKGROUND DOCUMENT

STATISTICAL METHODS OF RELIABILITY DETERMINATION

1.     INTRODUCTION

        The increased emphasis placed upon high reliability for Army missile
and rocket systems and the need for better collection, analysis, and reporting
of success and failure data has made the measurement of reliability that is
obtained throughout the Engineering Test Program one of the most important in-
dices used to assess missile system capability.  Reliability and its companion
index, accuracy, determined under various test conditions are two factors which
are relevant in any missile system evaluation.  The fundamental purpose of re-
liability is to estimate the probability of the successful performance of an
item under specified conditions and to compute the expected value of this prob-
ability with a specified confidence for the population of the items considered .

        A well planned and executed reliability program for engineering test
results in data that reduces to provide:

        a.  Numerical estimates of system probability of successful perfor-
mance under specified conditions.
        b.  The confidence or degree of assurance that is attained in the
estimates of a. above as measures of the true reliability.
        c.  The existence (or non-existence) of significant trends in reli-
ability as a function of test conditions.
        d.  Failed parts information.
        e.  Reliability estimates that are part of the factors used in a
system effectiveness evaluation.

        The statistical methods of reliability used in planning, collecting,
analysis, the reporting of reliability data for missile and rocket systems
during the Engineering Test program are discussed in the following sections.

2.     RELIABILITY REQUIREMENTS

        Missile system reliability requirements are stated in the following
ways:

        a.  The minimum inflight reliability of the missile shall be 0.90
against a specified target at a range of not less than _____ meters and
an altitude of _____ meters.
        b.  After a 6 (or 12) month storage the missile shall undergo a
functional checkout with a reliability of at least 0.95.
        c.  Missile prefire checkout reliability shall be at least 0.95.
        d.  The missile system availability[1] shall not be less than 0.90.
        e.  The tracking (or identification) radar shall have a Mean Time
Between Failure[2] (MTBF) of not less than _____ hours.

[1] & [2] See Glossary
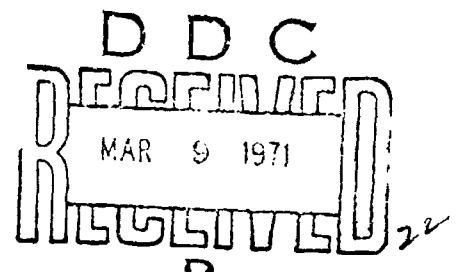
The Mean Time Between Failure (MTBF) is sometimes used as a descriptor of reliability when it is not appropriate to express the reliability in terms of a probability.

3.      TEST OBJECTIVES FOR RELIABILITY

In any laboratory or firing test to be conducted upon a missile system or subsystem which is to obtain, among other information, estimates of reliability the following questions need to be answered in the planning stage.

a.  Under what specific test conditions is the reliability to be measured?

b.  What probability distributions are applicable?  Normal (Gaussian)? Exponential?  Binomial? etc.

c.  Are the test conditions selected fixed for this particular experiment, or are they randomly chosen from a larger set of conditions?

d.  What are the chief sources of variation to be expected?

e.  How are the items (components, subsystem, missiles), to be allocated among the test conditions (treatments)?

f.  What particular statistical methods should be used and for what purpose?

g.  What precautions are necessary in testing so that the data are not invalidated for statistical analyses?

h.  How is a failure defined?[3]

i.  What constitutes a no-test category?[4]

When appropriate, an analysis should be carried out with dummy data to determine if the results are answering the right questions and if a more efficient reliability design can be used without loss of essential information.

4.      COLLECTION AND FORMAT OF DATA FOR RELIABILITY ANALYSES

a.  Missile reliability from preflight checkout and inflight performance data.  Tables I through V can be used for scorekeeping purposes according to the kind of data available.  The entries in the tables are the number of successes (S) and failures (F) scored for the various major assemblies in a missile  during flight (Table I), and similarly scored for missile flight performance during various phases of the trajectory (Table II).

Table III gives a breakdown of one definition of mission reliability as a function of other reliabilities.  Table IV is a combination scorekeeping of missile performance according to jamor assemblies, missile flight performance in various phases, and impact (intercept) reliability.

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[3] See Glossary
[4] See Glossary for possible conditions of no-test

Table I.  Missile Performance

| Round Number | Propulsion System | Hydraulics | Guidance | Power Supply | Structure |
|---|---|---|---|---|---|
| 1 | S | S | S | S | S |
| 2 | S | F | S | S | S |
| 3 | S | S | F | F | S |
| Reliability Estimate | $\dfrac{S'}{S' + F'}$ | $\dfrac{S'}{S' + F'}$ | $\dfrac{S'}{S' + F'}$ | $\dfrac{S'}{S' + F'}$ | $\dfrac{S'}{S' + F'}$ |

$S'$ = No. of successes     $F'$ = No. of failures

Table II.  Flight Performance

| Round No. | Launch-Boost | Mid-Course | Terminal |
|---|---|---|---|
| 1 | S | S | S |
| 2 | F | S | S |
| 3 | S | S | F |
| Reliability Estimate | $\dfrac{S'}{S' + F'}$ | $\dfrac{S'}{S' + F'}$ | $\dfrac{S'}{S' + F'}$ |

Table III.  Mission Reliability

| Mission Reliability | = | (Pre-Launch Rel.) x (Launch Rel.) x (Flight Rel.) x (Impact or Intercept Rel.) x (Warhead Rel.) |
|---|---|---|
| Pre-Launch Reliability | = | $\dfrac{\text{No. of Missiles that Pass Pre-Launch Checkout}}{\text{Total No. of Missiles Checked Out}}$ |
| Launch Reliability | = | $\dfrac{\text{No. of Missiles that Are Successfully Launched}}{\text{No. of Missiles that Pass Pre-Launch Checkout}}$ |
| Flight Reliability | = | $\dfrac{\text{No. of Missiles that Experienced No Known Subsystem Malfunction Which Would Lead to an Abort or a Large Deviation From the Flight Path}}{\text{No. of Missiles that are Successfully Launched}}$ |
| Intercept Reliability | = | $\dfrac{\text{No. of Missiles that Attained a Miss} \le \text{a Predetermined Value}}{\text{No. of Missiles that Experienced No Known Subsystem Malfunction Which Would Lead to an Abort or a Large Deviation From the Flight Path}}$ |

b. Dependence of reliability upon test conditions: Table V illustrates the format for collecting the data used in determining the effect of test conditions upon missile inflight reliability. The number of successful (S) and failed (F) missiles with respect to inflight reliability are tabulated according to a two way classification (here two ranges, $R_1$ and $R_2$, and three altitudes, $A_1 - A_3$, for example).

The data thus collected in this contingency table can be statistically analysed to determine if missile inflight reliability is significantly dependent upon these test conditions.

c. Reliability Tests on Subsystems or Components:

1) Functional checkouts

   a) Data logs (either discrete or continuous) are maintained which are complete and annotated. The environmental conditions under which the test(s) are conducted are specified, and any necessary changes in them from the original plan of test must be explicitly noted.

   b) Unusually large or small observations of the variable(s) being tested should be carefully noted. If specification limits are to be checked against, these limits (engineering or statistical) must be fully described and understood as to their meaning.[5]

2) Sensitivity Tests

   a) These tests are of the go - no go type; the item under test either operates or fails to operate under a given input or stimulus. The statistical analysis of the resulting data depends upon the objectives of the plan of test and how the data are to be collected.

   b) The Bruceton or up-and-down method is one of the statistical techniques most often used in analyzing sensitivity data.

3) Life Tests on Electronic Items

   a) For electronic items which are to be life tested for demonstrating whether or not their Mean Time Between Failure equals or exceeds specification limits.

   b) Electronic equipment undergoing tests to determine their

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[5] Limits of error or uncertainty allowed are often vaguely stated which leads to a misinterpretation of their meaning. See Reference A, Section 4, Chapter 23 for a full discussion of this subject.

Table IV. Use of Flight Data in Combined Tables

| Round Number | Missile Performance | | | | | Flight Performance | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | Motor | Hydraulics | Air-Borne Guidance | Power Supply | Structure | Launch-Boost | Mid-Course | Terminal Phase |
| 1 | S | S | S | S | S | S | S | F |
| 2 | S | S | F | S | S | S | S | F |
| 3 | S | S | F | S | S | S | F | S |
| 4 | S | S | S | S | S | S | F | S |
| Reliability Estimate | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ | $\frac{S'}{S'+F'}$ |

S' = No. of Successes

F' = No. of Failures

NOTE: The success ratios (reliability estimates) are merely point or sample values of the true but unknown population reliability. What can be said about the latter is stated in the form of confidence limits (see Glossary).

Table IVa. Numerical Example

| Round Number | Total Inflight Performance | P% Lower Confidence Limit on the Population Reliability | | Corresponding Two Sided Confidence Limits |
|---|---|---|---|---|
| | | Confidence Level (P%) | Lower Con- fidence Limit | |
| 1 | S | | | |
| 2 | S | | | |
| 3 | S | | | |
| 4 | S | 50 | 0.77 | (0.66, 0.96) |
| 5 | F | 75 | 0.66 | (0.57, 0.98) |
| 6 | S | 90 | 0.55 | (0.45, 0.993) |
| 7 | S | 95 | 0.48 | (0.42, ).996) |
| RELIABI- LITY ESTIMATE | 6/7 = 0.86 | | | |

NOTE: Even if all rounds were declared inflight successes, the 50% lower confidence limit on the population reliability would only be 0.906. The 95% lower limit is 0.65.

Table IVb. Target Reliability

| No. of Rounds Fired | Number of Successful Rounds* | Reliability |
|---|---|---|
| N | X | $\dfrac{X}{N}$ |

* Number of rounds with a miss-distance less than or equal to a specified quantity

If a hit reliability is to be estimated for a surface-to-surface missile, then such a reliability can be found from the ratio:

$$\dfrac{\text{Number of Missiles Hitting the Target}}{\text{Number of Successfully Launched Missiles}}$$

Table V.  Number of Successful and Failed Missiles
At Two Ranges and Three Altitudes

| Target Altitude | Target Range | | | | Totals | |
|---|---|---|---|---|---|---|
| | $R_1$ | | $R_2$ | | | |
| | S | F | S | F | S | F |
| $A_1$ | | | | | | |
| $A_2$ | | | | | | |
| $A_3$ | | | | | | |
| TOTALS | | | | | | |

Figure 1 below shows an example of the format of the data that are collected for testing the voltage from the firing point at which some squibs are activated.

| Voltage from Firing Point | SQUIB NUMBER | | | | | | | | | | 0 - no fire X - fire | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | etc | ... | ... | ... |
| 0.6<br>0.7<br>0.8*<br>0.9<br>1.0<br>1.1 | X | 0 | X | X | 0 | X | | ... | X | 0 | ... | X | 0 |

*  Initial voltage at which approximately one-half of the squibs will be
   activated.

Figure 1.  Fire, No-Fire Results for Squibs

To find the least value of the input voltage with which say, 0.99% of the squibs can be expected to fire at a certain confidence level is one of the objectives of this type of test.  Several types of statistical analyses are used, depending upon the experimental objectives and the test set-up.

reliability and/or availability require data of the
following type:

(1) Accurate and complete time to failure readings.
(2) The total number of failures observed.[8]

Table VI shows the format to be used for collecting further data on
this type of test.

Table VI. Number of Squibs That Fired and Didn't Fire at Various
Voltages from Firing Point and Percent of Fired Squibs

| Voltage From Firing Point | Squibs Fired | Squibs That Didn't Fire | Percent of Fired Squibs |
|---|---|---|---|
| 0.0<br>0.5<br>0.6<br>0.7<br>0.8<br>0.9<br>1.0<br>1.1 | | | |

## 5. DESIGN FOR RELIABILITY

### 5.1 RELIABILITY TEST PLAN

### 5.1.1 Reliability in Engineering Tests

#### 5.1.1.1 The Types of Data to be Collected

a. Attribute or discrete data·

1) Number of go-no go (success and failure) counts in the
following types of testing.

a) Ground equipment checkouts.
b) Missile prefire checkout.
c) Inflight phase of missile.
d) Launch phase of missile.
e) Sensitivity tests on squibs, S_ devices; drop tests.
f) Reliability (life) tests on electronic equipment.[7]

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[8] A failure must be clearly defined.
[7] Section II contains more information on the statistics of life testing.

In all the tests enumerated above a concise definition of a successful operation or mission is needed for the item under test. Also, the criteria must be determined for declaring which events constitute a "no test" category.

  b. Variable or continuous data.

    1) Time readings.

      a) Operating times on items.
      b) Down times due to maintenance actions.
      c) Arming times on S&A devices.
      d) Measurements of time as an independent variable.
      e) Ignition and burnout times of propulsion systems undergoing specific tests.

    2) Linear or angular measurements (examples).

      a) Azimuth and pitch.
      b) Arming distance of S&A
      c) Drifts in sensing devices.

    3) Other physical measurements.

      a) Pounds - thrust.
      b) Impulse.
      c) Temperature.
      d) Power, volts, etc.

    4) Data comprising several parameters whose effect on a dependent variable is considered when one of the former is held fixed; e.g., the burst point of a warhead on a missile is a function of position, velocity, fuzing, etc. What is the effect of a constant velocity on the reliability of a warhead burst point?

5.1.1.2 Uses of Collected Data for a Reliability Analysis

  a. From Attribute data, (go, no-go or success-failure type of data).

    1) Reliability estimates of ground checkout equipment, missile checkout. Confidence limits on the true reliability.
    2) Reliability estimates of launch and inflight phases of the missile with corresponding confidence limits.
    3) Reliability and safety of S&A devices as a function of distance from armed position.
    4) Verify with a prescribed confidence that the reliability of an item for a given time is at least a specified value.
    5) Determine if missile reliability depends significantly upon test conditions (e.g., target types, target range) or specified environmental conditions (e.g., missile fired after a cold temperature treatment and/or shock treatment).

6) Estimate of non-system failure ratio, i.e., the proportion of failures that cannot be attributed to any system malfunction. For example, the holds on missile firings due to range instrumentation delays, range safety, etc.

The success/failure data collected is evaluated probability wise by using the binomial distribution function or the chi-square statistic.

b. From Variable Data.

1) Estimates of a parameter which depends upon one or more variables. Regression analysis used.
2) Statistical tolerance limits to verify that performance limits are met (or reject an item not within specifications).
3) Availability and maintainability estimates from down time and Mean Time to Failure (MTTF) data.
4) Identification of large biases (or real effects) in random data.

5.1.2       Reliability Demonstrations Tests.[8]

5.1.2.1    The Types of Data to be Collected.

a. The number of failures observed on the item(s) undergoing test.
b. The operational life of the item(s) on test.
c. The time at which the test is to be terminated.
d. The number of observed failures allowed on the test.
e. The total number of items on test.

The life test performed under condition c. is called a truncated life test and under condition d., a censored life test.[9]

5.1.2.2    The Requirements or Specifications in a Life Test May be of the Following Types:

a. The minimum acceptable Mean Time Between Failure (MTBF) with the associated confidence level.
b. The specified MTBF and a minimum acceptable MTBF with associated producer's and buyer's risks.[10]   These are specifications for a sequential life test.[11]
c. The specified MTBF ($\theta_0$) and two values of the MTBF, $\theta_1 = 0.80 \ \theta_0$ and $\theta_2 = 1.20 \ \theta_0$. Also, the producer's and buyer's risks.[12]

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[8] In particular, life tests conducted in laboratories on electronic items.
[9] Reference E contains a full discussion on the statistics used in planning and evaluating life test data.
[10] See Glossary
[11] See Glossary
[12] WSMR Regulation 715-6, Appendix II, contains an example with these specifications. The AMC Reliability Handbook, AMCP 702-3, October 1968, also contains examples on life tests. See pages F-44 through F-56 for sequential test information.

It is assumed that the time to failure on many electronic items undergoing life test follows an exponential probability distribution. [13] Reference E makes this assumption in all of the statistical methods. There are however, times to failure experienced by some items on test that follow different probability laws (e.g., the Weibull distribution), and for a valid reliability analysis of life test data the proper distribution functions must be ascertained either from experience files or a suitable analysis of the data.[14]

6.        DATA REDUCTION AND PRESENTATION

6.1       FROM ATTRIBUTE (DISCRETE) DATA

        a.  Reliability of Missile During Preflight Checkout Procedures

        If missile automatic checkout equipment is used, and X is the number of "go" conditions obtained from a total of N discrete tests, then X/N is the reliability estimate of the missile during the prefire checkout. The data collected for this purpose must be unambiguous and complete to be of any use. The output data must eigher measure the "go" or "no go" condition of the item under test and not be compounded with other effects.

        b.  Inflight Reliability of the Missile.

        The inflight reliability estimate of the missile hinges upon the definition of a successful missile during this phase of the missile trajectory. An example of such a definition is given in Table III, (flight reliability).

        In all such cases where the ratio of the number of successes to the number of "trails" is computed, we have a "point" estimate of the reliability. How good this point estimate is, or how consistent it is with respect to the true but unknown reliability, is discussed in paragraph 6.1.g. (confidence limits on reliability).

        Data which can be planned for, collected, and summarized as in Tables I - IV yield these point estimates of reliability. These estimates can be used separately or in conjunction with each other; the latter use will require further examination and is discussed in paragraph d below.

        c.  Statistical Analysis for Contingency Tables.

        The data collected on missile inflight successes and failures with respect to various test conditions are analyzed with regard to the calculated expected number of successes and failures. A comparison is made between these expected values and the actual values to see if they are compatible under the assumption that missile inflight reliability is not dependent upon the test

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[13] See Glossary

[14] Reference E contains several statistical tests used to determine if the failure times are exponentially distributed.

conditions stated. The Chi-square ( $^2$) statistic is used in this analysis.[15]

      d. Reliability as the Product of Subsystem (or Component).
         Reliabilities, $R = r_1\ r_2\ .\ .\ .\ r_k$

If reliability is to be computed as the product of two or more component reliabilities, the following precautions are necessary.

      1) The components must be statistically independent of each other, i.e., the failure of one component does not cause another component to fail and the components are not sensitive to the same or to correlated environmental stresses.[16]

      2) It is assumed that if one component fails the system fails (components in series in contrast to parallel).

During the missile system engineering test and evaluation program, application of this product rule for reliability is not always valid, and when it is used care must be exercised to see that the above assumptions are reasonably satisfied.

      e. Hit Probability

      $H = AR,$

where     $A$ = Accuracy of missile at target measured in terms of the probability of the missile hitting a target of prescribed size,

and      $R$ = Missile inflight reliability, or the ratio of the number of rounds that hit the target area without an observed malfunction to the number of successfully launched missiles.

Such a probability as defined above is a conditional probability, i.e., the accuracy is determined upon the condition that only the inflight successful rounds are counted in the accuracy calculation. It will be noticed here that whereas R is a discrete measurement, A is not (a variable or continuous measure). Compare with the definition of hit reliability (below Table IV).

The three estimates - target reliability, hit reliability, and hit probability are all values that can be computed, depending upon the definition of the term reliability in a specific program.

      f. Graphical Representation of the Accumulative Reliability

      1) The Accumulative Reliability

         The accumulative missile reliability in a series of firings is based upon the accumulated proportions of successes to

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[15] Reference A, Section 2, Chapters 8 and 9.
[16] See Reference F, pp. 1-6.

the sum of successes and failures at each time of firing.
Table VII and Figure 2 illustrate the method.

2) Moving Average Graph of Missile Reliability

The equally weighted moving average reliability graph of
N points (N an odd integer) is based upon the average accumu-
lative reliability of (a) preceding $(\frac{N-1}{2})$ missiles, (b) the
"midpoint" missile, and (c) the succeeding $(\frac{N-1}{2})$ missiles.
This average or smoothed reliability is computed at the mid-
point, $R_0$, according to the formula

$$R_0 = \frac{1}{N} \sum_{i=-\frac{N-1}{2}}^{\frac{N-1}{2}} R_i$$

If the resulting values are plotted upon the same type of graph as
appears in Figure 2, the points corresponding to the accumulative reliability
of the first $(\frac{N-1}{2})$ rounds and the last $(\frac{N-1}{2})$ rounds are excluded.
Depending upon the total number of rounds to be considered, N may be 5, 7, 9,
or larger.

The purpose of this type of graph is to smooth out the large short
time fluctuations in missile reliability and identify trends (if they exist)
in reliability over longer intervals of time.

g. Confidence Limits on Reliability

A point estimate of reliability is seldom enough information about
the reliability just by its self. Confidence limits provide a measure of how
good that estimate is or how "reliable" it is. Associated with a confidence
limit is the confidence level or confidence coefficient which is the degree of
assurance made about the confidence limits.
Thus if a reliability estimate is

$$R = \frac{5}{10} = 0.50, \text{[17]}$$

the true reliability, R, is at least 0.50 with 38% confidence, i.e., the fidu-
cial (confidence) probability that the true reliability is at least 0.50 is
38%. Stated in an equation

$$\text{Prob } [ R \geq 0.30 ] = 0.38$$

Likewise, Prob $[ R \geq 0.30 ] = 0.85$. These values, 0.50 and 0.30, are both

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[17]  See Reference G.

Table VII.  Accumulative Missile Reliability

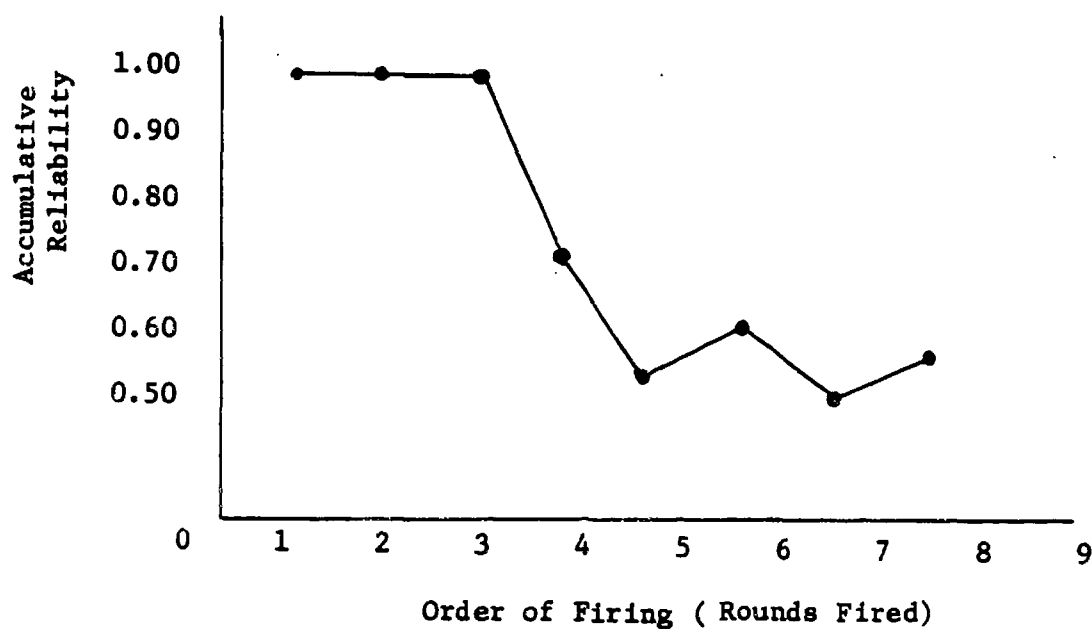| Round Number | Score (Success or Failure) | Reliability Estimate |
|---|---|---|
| 1 | S | 1.00 |
| 2 | S | 1.00 |
| 3 | S | 1.00 |
| 4 | F | 0.75 (3S, 1F) |
| 5 | F | 0.60 (3S, 2F) |
| 6 | S | 0.67 (4S, 2F) |
| 7 | F | 0.57 (4S, 3F) |
| 8 | S | 0.63 (5S, 3F) |
| - | - | - |
| - | - | - |
| - | - | - |



Figure 2.   Accumulative Missile Reliability Graph

lower confidence limits on reliability, the first at the 38% confidence level and the second at the 85% level. Upper confidence limits can be given as well, although the lower limits are more appropriate in many cases (e.g., reliability requirements are most often stated in terms of minimum values).[18]

Confidence limits on a product of reliabilities can often be approximated with sufficient accuracy. Whenever this can be done, such limits are an essential part of the reliability analysis.[19]

      h.  Relationships Between Confidence Limits on Reliability, Confidence Levels, the Number of Failures, and the Sample Size.

There are three methods for determining any one of these factors if the others are knows (discrete data assumed):

    1)  By published tables.[20]
    2)  By graphs[21]
    3)  By special slide rules.[22]

This information is used to construct tables of predicted reliabilities assuming different sample sizes, confidence levels, and the number of failures.

6.2      FROM CONTINUOUS (VARIABLE DATA)

Reliability estimation (and the computation of confidence limits) based upon continuous data involves two parameters - the reliability itself (or availability) and the Mean Time Between Failure (MTBF). The fundamental formulas, assuming an exponential distribution of life-times,[23] are

      a.  Reliability = R = probability of no failure in time t

$$= e^{-t/\theta}$$

where   $\theta$ = MTBF

      b.  Availability = $\dfrac{MTBF}{MTBF + MDT}$

where   MDT = Mean Down Time

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[18]  Lower confidence limits on the reliability thus provide a means of determining whether or not missile reliability requirements have been met.

[19]  One such procedure is outlined in 8 below.

[20]  Reference K.

[21]  See Reference G.

[22]  A confidence Limit Computer, Sandia Corp. Reliability Department, Sandia Corp; Albuquerque, New Mexico, General Dynamics Corp. Pomona, California, also has prepared a useful reliability sliderule.

[23]  If the life-times are Weibull Distributed, see Reference B, pages F-7 through F-10

Here, availability is defined as the probability that an equipment will be operational at a random point in time. The MDT may or may not include down time due to supply and administrative delays.

7.        RELIABILITY AND SAFETY

7.1      PROBLEMS

Reliability enters missile safety problems in three ways:

        a.   Finding the probability of an unsafe condition (e.g., the event that a rocket motor will prematurely fire because of improper ignition time, or a chamber rupture due to excessive pressure).
        b.   Finding the probability of a surface-to-surface missile hitting within a designated "friendly" area.
        c.   To determine the limits of a safety region or the probability of a hit within such a region.

7.2      DATA COLLECTION

Data collected are of the following types.

        a.   Ignition times of rocket motors with corresponding other data (e.g., thrust, impulse, chamber pressure).
        b.   Boundary values of the friendly area and its centroid; the dispersion (variance) values of the missile in flight. Geometry of the zone of approach of the missile to the area in question and the nominal missile impact point.
        c.   Number of rounds to be considered and their coordinates of impact (range, deflection). The geometrical form of the safety region and the location (within or outside of the region) of prime points of sub-areas of interest.

7.3      DATA REDUCTION

No standard set of formulas can be given that can be applied in general to every case mentioned above. Each reliability safety problem calls for a particular mode of attack in its solution. The probability distributions associated with the relevant data have to be first determined (and they may not be known in their entirety).

A decision to use variable or discrete data in the analysis, and the sample sizes of items to be tested for safety purposes (with the corresponding high probability levels of no failure) are part of the plan of test for reliability and safety.

If the safety test is to be conducted on items using the attributes criterion (success or failure), exhorbitantly large sample sizes are required to satisfy very low probabilities of failure. Testing by variables utilizes much smaller sample sizes in addition to providing other information, e. g., the effect of environmental factors upon the item under test.

8.      FORMULAS FOR FINDING LOWER CONFIDENCE LIMIT ON A PRODUCT OF RELIABILITIES

The formula for n (8.2) gives sufficiently accurate results for large sample sizes.  It is based upon the use of propagation of error formula,

$$\sigma^2(F) = \sum_{i=1}^{k} \left(\frac{2F}{2\chi_2}\right)^2 \sigma^2(\chi_i)$$

where $F = F(X_1, X_2, \ldots, X_k)$.

8.1      SYMBOLS USED

Let $\hat{R} = \hat{P1}\ \hat{P2}\ \ldots\ \hat{Pk}$

where          $\hat{R}$ = estimated reliability,

$P_i = \frac{s_i}{n_i}$          $(i = 1, 2, \ldots, k)$

= the $i^{th}$ component estimated reliability,

and          $s_i$ = number of successes in $n_i$ independent tests.

Find the lower confidence limit, $R_L$, on the true reliability, R, such that

Prob $[R \geq R_L] = 0.90$

where 0.90 is the prescribed confidence level.

8.2      FORMULAS

Compute:

$$\sum_{i}^{k} \frac{n_i}{s_i} - k$$

$$n = \frac{1}{\sum_{1}^{k} \frac{1}{s_i} - \sum_{1}^{k} \frac{1}{n_i}}$$

and put $r = \hat{R}n$

where r = adjusted value to be used in Reference I. Interpolating within the tables of the above reference (for the computed values of n and r), the value of $R_L$ is obtained.

If Reference E can be obtained, Table 4(c), p. 3.85, is then utilized to quickly obtain the desired value of $R_L$. In using this table, the column headings of numbers are, for this particular problem, values of (n-r) and the row heading numbers are values of n.

(EXAMPLE)

Suppose

$$R = \frac{17}{18} \times \frac{20}{21} \times \frac{11}{15} = \frac{3740}{5670} = 0.66$$

Then

$$n = \frac{3.472 - 3}{0.1997 - 0.1698} = \frac{0.472}{0.0299} = 15.8$$

and

$$r = 10.42.$$

Using Reference ·1 with (n-r) = 5.4 and n - 15.8, we have $R_L$ = 0.47, the interpolated value.

9.  THE RELIABILITY FORMULA FOR LIFE-TEST DATA THAT FOLLOW A WEIBULL DISTRIBUTION

It is assumed that the sample failure-times have been satisfactorily checked as to their being from a Weibull population.

9.1  SYMBOLS USED

Let t = the failure-free time, or mission time, of the equipment,

$\eta$ = (eta), the scale parameter of the Weibull probability distribution, and

$\beta$ = the shape parameter of this distribution.

9.2  FORMULA

Reliability = Prob [no failure in time t]

$$= \exp\left[-\left(t/\eta\right)^{\beta}\right], \quad \beta \neq 1$$

If $\beta < 1$, the failure rate is relatively high in early life, decreasing with

passing time. If $\beta > 1$, the converse is true. If $\beta = 1$, the Weibull distribution reduces to the exponential distribution of life-times. $\beta = 1$ corresponds to a constant failure rate, i.e., independent of time, when the failures of the item(s) under test occur in a random manner - after the debugging period and before wearout of the item.

The scale and shape parameters, $\eta$ and $\beta$ respectively, can be estimated from life-test data that follow the Weibull distribution.[24]

Also, tests are available for determining whether or not life-test data obey sufficiently well this distribution.[25]

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[24] See Reference J.
[25] See Reference B.

GLOSSARY

1. Availability:

   a. Pointwise availability is the probability that an item will be operational at any given time.

   b. Interval availability is the proportion of time during which an item is operational, e.g., 23 out of every 24 hours for a search radar.

2. Buyers Risk: The chance the buyer (or consumer) assumes in accepting a substandard product, usually designated by the Greek letter $\beta$ and numerically equal to a probability. In the statistics of life testing, $\beta$ is the probability of accepting an item with a MTBF less than the minimum required value.

3. Confidence Lim t, Lower: The least value that the true reliability (or MTBF) can assume with a specified degree of assurance, assuming that reliability (MTBF) estimates are obtained from similar items in repeated tests under the same conditions.

   Stated in the form of an equation, if

   $$\text{Prob} \quad [\, R \geq R_L \,] = P\%,$$

   then  $R$ = true but unknown reliability,

   $R_L$ = the $P\%$ lower confidence limit on R,

   . and  $P\%$ = the confidence level (degree of assurance) associated with the limit or bounds, $R_L$.

   $P\%$ usually has the value of 95 or 90.

4. Exponential Distribution: A probability function of the form $1 - e^{-t/\theta}$, which gives the probability that an item will fail in time t if $\theta$ is the Mean Time Between Failure. (See definition of MTBF.) The reliability function is $e^{-t/\theta}$.

5. Mean Time Between Failure (MTBF): The average number of hours of life per failure calculated from the equation

   $$\text{MTBF} = \frac{\text{Total Operating Time}}{\text{Total Number of Failures}}$$

   The failure rate is then the reciprocal of the MTBF.

6. No-Test: The conditions under which an item is not counted in reliability scorekeeping procedures. Possible conditions for missile system tests are: (1) Drone malfunction after missile launch. (2) Missile forced beyond its

design limits. (3) Operator errors if only missile hardware capability is being scored. (4) Operations which are not completed in the preflight period to the desired point due to conditions external to the system or where no data exists on the completions.

7. <u>Producers Risk</u>: The chance the producer assumes in rejecting an item that meets or exceeds stated requirements, usually designated by the Greek letter $\alpha$ and numerically equal to a probability.

8. <u>Reliability</u>: The probability that an item will successfully perform its intended mission under stated conditions. Interval reliability is defined as the product of pointwise availability and reliability, i.e.,

     Prob    [ item is operational at any time $t_o$ ]

  x    Prob    [ item's successful performance during the
                 time interval $t_1 - t_o$ ]

9. <u>Sequential Life Test</u>: A test which proceeds until a decision is reached to either accept or reject the item(s) under test according to criteria established in the design of the experiment. Sequential analysis is a statistical technique that provides the criteria, procedure, and mathematics for this type of test.

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